Paper 1: Pure Mathematics Mark Scheme

Question	Scheme	Marks	AOs
1 **Way 1**	Uses $y = mx + c$ with both (3, 1) and (4, −2) and attempt to find m or c	M1	1.1b
	$m = -3$	A1	1.1b
	$c = 10$ so $y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or Way 2	Uses $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$ with both (3, 1) and (4, −2)	M1	1.1b
	Gradient simplified to -3 (may be implied)	A1	1.1b
	$y = -3x + 10$ o.e.	A1	1.1b
		(3)	
Or Way 3	Uses $ax + by + k = 0$ and substitutes both $x = 3$ when $y = 1$ and $x = 4$ when $y = -2$ with attempt to solve to find a, b or k in terms of one of them	M1	1.1b
	Obtains $a = 3b$, $k = -10b$ or $3k = -10a$	A1	1.1b
	Obtains $a = 3$, $b = 1$, $k = -10$ Or writes $3x + y - 10 = 0$ o.e.	A1	1.1b
		(3)	
		(7 marks)	

Notes:

M1: Need correct use of the given coordinates
A1: Need fractions simplified to -3 (in ways 1 and 2)
A1: Need constants combined accurately

N.B. Answer left in the form $(y - 1) = -3(x - 3)$ or $(y - (-2)) = -3(x - 4)$ is awarded M1A1A0 as answers should be simplified by constants being collected

Note that a correct answer implies all three marks in this question

Question	Scheme	Marks	AOs
2	Attempt to differentiate	M1	1.1a
	$\frac{dy}{dx} = 4x - 12$	A1	1.1b
	Substitutes $x = 5 \Rightarrow \frac{dy}{dx} = ...$	M1	1.1b
	$\Rightarrow \frac{dy}{dx} = 8$	A1ft	1.1b
		(4 marks)	

Notes:

M1: Differentiation implied by one correct term

A1: Correct differentiation

M1: Attempts to substitute $x = 5$ into their derived function

A1ft: Substitutes $x = 5$ into **their** derived function **correctly** i.e. Correct calculation of their $f'(5)$ so follow through slips in differentiation

Question	Scheme	Marks	AOs
3(a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1	1.1b
	$\overrightarrow{AB} = 5\mathbf{i} + 10\mathbf{j}$	A1	1.1b
		(2)	
(b)	Finds length using 'Pythagoras' $\lvert AB \rvert = \sqrt{(5)^2 + (10)^2}$	M1	1.1b
	$\lvert AB \rvert = 5\sqrt{5}$	A1ft	1.1b
		(2)	
		(4 marks)	

Notes:

(a)

M1: Attempts subtraction but may omit brackets

A1: cao (allow column vector notation)

(b)

M1: Correct use of Pythagoras theorem or modulus formula using their answer to (a)

A1ft: $\lvert AB \rvert = 5\sqrt{5}$ ft from their answer to (a)

Note that the correct answer implies M1A1 in each part of this question

Question	Scheme	Marks	AOs
4(a)	States or uses $f(+3) = 0$	M1	1.1b
	$4(3)^3 - 12(3)^2 + 2(3) - 6 = 108 - 108 + 6 - 6 = 0$ and so $(x - 3)$ is a factor	A1	1.1b
		(2)	
(b)	Begins division or factorisation so x $4x^3 - 12x^2 + 2x - 6 = (x - 3)(4x^2 + \ldots)$	M1	2.1
	$4x^3 - 12x^2 + 2x - 6 = (x - 3)(4x^2 + 2)$	A1	1.1b
	Considers the roots of their quadratic function using completion of square or discriminant	M1	2.1
	$(4x^2 + 2) = 0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2 + 2 > 0$ for all x So $x = 3$ is the only real root of $f(x) = 0$ *	A1*	2.4
		(4)	
			(6 marks)

Notes:

(a)

M1: States or uses $f(+3) = 0$

A1: See correct work evaluating and achieving zero, together with correct conclusion

(b)

M1: Needs to have $(x - 3)$ and first term of quadratic correct

A1: Must be correct – may further factorise to $2(x - 3)(2x^2 + 1)$

M1: Considers their quadratic for no real roots by use of completion of the square or consideration of discriminant then

A1*: A correct explanation

Question	Scheme	Marks	AOs
5	$\text{f}(x) = 2x + 3 + 12\,x^{-2}$	B1	1.1b
	Attempts to integrate	M1	1.1a
	$\int\left(+2x+3+\dfrac{12}{x^2}\right)\text{d}x \;=\; x^2+3x-\dfrac{12}{x}$	A1	1.1b
	$\left((2\sqrt{2})^2+3(2\sqrt{2})-\dfrac{12(\sqrt{2})}{2\times 2}\right)-(-8)$	M1	1.1b
	$=16+3\sqrt{2}$ *	A1*	1.1b
			(5 marks)

Notes:

B1: Correct function with numerical powers
M1: Allow for raising power by one. $x^n \to x^{n+1}$
A1: Correct three terms
M1: Substitutes limits and rationalises denominator
A1*: Completely correct, no errors seen

Question	Scheme	Marks	AOs
6	Considers $\dfrac{3(x+h)^2-3x^2}{h}$	B1	2.1
	Expands $3(x+h)^2 = 3x^2 + 6xh + 3h^2$	M1	1.1b
	So gradient $= \dfrac{6xh+3h^2}{h} = 6x+3h$ or $\dfrac{6x\delta x+3(\delta x)^2}{\delta x} = 6x+3\delta x$	A1	1.1b
	States as $h \to 0$, gradient $\to 6x$ so in the limit derivative $= 6x$ *	A1*	2.5
		(4 marks)	
Notes:			

B1: Gives correct fraction as in the scheme above or $\dfrac{3(x+\delta x)^2-3x^2}{\delta x}$

M1: Expands the bracket as above or $3(x+\delta x)^2 = 3x^2+6x\delta x+3(\delta x)^2$

A1: Substitutes correctly into earlier fraction and simplifies

A1*: Uses Completes the proof, as above (may use $\delta x \to 0$), considers the limit and states a conclusion with no errors

Question	Scheme	Marks	AOs
7(a)	$\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6.\left(-\frac{x}{2}\right)+\binom{7}{2}2^5.\left(-\frac{x}{2}\right)^2 + ...$	M1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = 128 + ...$	B1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = ... -224x + ...$	A1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = ... + ... + 168x^2\ (+ ...)$	A1	1.1b
		(4)	
(b)	Solve $\left(2-\frac{x}{2}\right) = 1.995$ so $x = 0.01$ and state that 0.01 would be substituted for x into the expansion	B1	2.4
		(1)	
		(5 marks)	

Notes:

(a)

M1: Need correct binomial coefficient with correct power of 2 and correct power of x. Coefficients may be given in any correct form; e.g. 1, 7, 21 or ${}^7C_0$, ${}^7C_1$, ${}^7C_2$ or equivalent

B1: Correct answer, simplified as given in the scheme

A1: Correct answer, simplified as given in the scheme

A1: Correct answer, simplified as given in the scheme

(b)

B1: Needs a full explanation i.e. to state $x = 0.01$ and that this would be substituted and that it is a solution of $\left(2-\frac{x}{2}\right) = 1.995$

Question	Scheme		Marks	AOs
8(a)	**Way 1** Finds third angle of triangle and uses or states $\dfrac{x}{\sin 60°} = \dfrac{30}{\sin"50°"}$	**Way 2** Finds third angle of triangle and uses or states $\dfrac{y}{\sin 70°} = \dfrac{30}{\sin"50°"}$	M1	2.1
	So $x = \dfrac{30\sin 60°}{\sin 50°}$ $(= 33.9)$	So $y = \dfrac{30\sin 70°}{\sin 50°}$ $(= 36.8)$	A1	1.1b
	Area = $\frac{1}{2}\times 30\times x\times \sin 70\ °$ or $\frac{1}{2}\times 30\times y\times \sin 60$		M1	3.1a
	$= 478\ \text{m}^2$		A1ft	1.1b
			(4)	
(b)	Plausible reason e.g. Because the angles and the side length are not given to four significant figures Or e.g. The lawn may not be flat		B1	3.2b
			(1)	
			(5 marks)	

Notes:

(a)

M1: Uses sine rule with their third angle to find one of the unknown side lengths

A1: Finds expression for, or value of either side length

M1: Completes method to find area of triangle

A1ft: Obtains a correct answer for their value of x or their value of y

(b)

B1: As information given in the question may not be accurate to 4sf or the lawn may not be flat so modelling by a plane figure may not be accurate

Question	Scheme	Marks	AOs
9	Uses $\sin^2 x = 1 - \cos^2 x \Rightarrow\ 12(1-\cos^2 x) + 7\cos x - 13 = 0$	M1	3.1a
	$\Rightarrow 12\cos^2 x - 7\cos x + 1 = 0$	A1	1.1b
	Uses solution of quadratic to give $\cos x =$	M1	1.1b
	Uses inverse cosine on their values, giving two correct follow through values (see note)	M1	1.1b
	$\Rightarrow x =\ 430.5°,\ 435.5°$	A1	1.1b
			(5 marks)

Notes:

M1: Uses correct identity

A1: Correct three term quadratic

M1: Solves their three term quadratic to give values for cos x. (The correct answers are $\cos x = \frac{1}{3}$ or $\frac{1}{4}$ but this is not necessary for this method mark)

M1: Uses inverse cosine on their values, giving two correct follow through values - may be outside the given domain

A1: Two correct answers in the given domain

Question	Scheme	Marks	AOs
10	Realises that $k = 0$ will give no real roots as equation becomes $3 = 0$ (proof by contradiction)	B1	3.1a
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1	2.4
	$4k(4k-3) < 0$ with attempt at solution	M1	1.1b
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leqslant k < \frac{3}{4}$ *	A1*	2.1
			(4 marks)

Notes:

B1: Explains why $k = 0$ gives no real roots

M1: Considers discriminant to give quadratic inequality – does not need the $k \neq 0$ for this mark

M1: Attempts solution of quadratic inequality

A1*: Draws conclusion, which is a printed answer, with no errors (dependent on all three previous marks)

Question	Scheme	Marks	AOs
11 (a) Way 1	Since x and y are positive, their square roots are real and so $(\sqrt{x}-\sqrt{y})^2 \geq 0$ giving $x - 2\sqrt{x}\sqrt{y} + y \geq 0$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \dfrac{x+y}{2}$ *	A1*	2.2a
		(2)	
Way 2 Longer method	Since $(x-y)^2 \geq 0$ for real values of x and y, $x^2 - 2xy + y^2 \geq 0$ and so $4xy \leq x^2 + 2xy + y^2$ i.e. $4xy \leq (x+y)^2$	M1	2.1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \dfrac{x+y}{2}$ *	A1*	2.2a
		(2)	
(b)	Let $x = -3$ and $y = -5$ then LHS = $\sqrt{15}$ and RHS= -4 so as $\sqrt{15} > -4$ result does not apply	B1	2.4
		(1)	
		(3 marks)	

Notes:

(a)

M1: Need two stages of the three stage argument involving the three stages, squaring, square rooting terms and rearranging

A1*: Need all three stages making the correct deduction to achieve the printed result

(b)

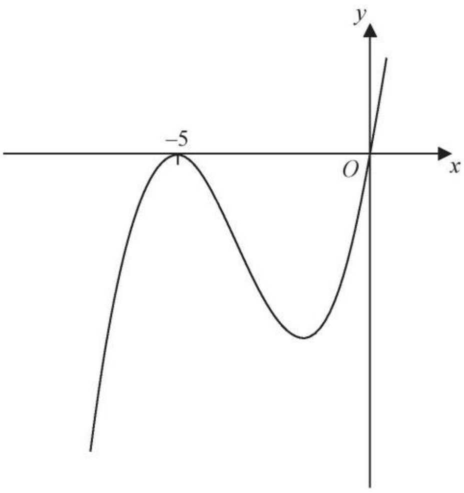
B1: Chooses two negative values and substitutes, then states conclusion

Question	Scheme		Marks	AOs
12(a)	$2^{2x} + 2^4$ is wrong in line 2 - it should be $2^{2x} \times 2^4$		B1	2.3
	In line 4, 2^4 has been replaced by 8 instead of by 16		B1	2.3
			(2)	
(b)	**Way 1:** $2^{2x+4} - 9(2^x) = 0$ $2^{2x} \times 2^4 - 9(2^x) = 0$ Let $2^x = y$ $16y^2 - 9y = 0$	**Way 2:** $(2x+4)\log 2 - \log 9 - x\log 2 = 0$	M1	2.1
	$y = \frac{9}{16}$ or $y = 0$ So $x = \log_2\left(\frac{9}{16}\right)$ or $\dfrac{\log\left(\frac{9}{16}\right)}{\log 2}$ o.e. with no second answer	$x = \dfrac{\log 9}{\log 2} - 4$ o.e.	A1	1.1b
			(2)	
				(4 marks)

Notes:

(a)
B1: Lists error in line 2 (as above)
B1: Lists error in line 4 (as above)

(b)
M1: Correct work with powers reaching this equation
A1: Correct answer here – there are many exact equivalents

Question	Scheme	Marks	AOs
13(a)	$x^3+10x^2+25x = x(x^2+10x+25)$	M1	1.1b
	$= x(x+5)^2$	A1	1.1b
		(2)	
(b)	 A cubic with correct orientation	M1	1.1b
	Curve passes through the origin (0, 0) and touches at (−5, 0) (see note below for ft)	A1ft	1.1b
		(2)	
(c)	Curve has been translated a to the left	M1	3.1a
	$a = -2$	A1ft	3.2a
	$a = 3$	A1ft	1.1b
		(3)	
		(7 marks)	

Notes:

(a)
M1: Takes out factor x
A1: Correct factorisation – allow $x(x + 5)(x + 5)$

(b)
M1: Correct shape
A1ft: Curve passes through the origin (0, 0) and touches at (−5, 0) – allow follow through from incorrect factorisation

(c)
M1: May be implied by one of the correct answers for a or by a statement
A1ft: ft from their cubic as long as it meets the x-axis only twice
A1ft: ft from their cubic as long as it meets the x-axis only twice

Question	Scheme		Marks	AOs
14(a)	$\log_{10} P = mt + c$		M1	1.1b
	$\log_{10} P = \frac{1}{200}t + 5$		A1	1.1b
			(2)	
(b)	**Way 1:** As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$	**Way 2:** As $\log_{10} P = \frac{t}{200} + 5$ then $P = 10^{\left(\frac{t}{200}+5\right)} = 10^5 10^{\left(\frac{t}{200}\right)}$	M1	2.1
	$\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$	$a = 10^5$ or $b = 10^{\left(\frac{1}{200}\right)}$	M1	1.1b
	So a = 100 000 **or** b = 1.0116		A1	1.1b
	Both a = 100 000 **and** b = 1.0116 (awrt 1.01)		A1	1.1b
			(4)	
(c)(i)	The initial population		B1	3.4
(c)(ii)	The proportional increase of population each year		B1	3.4
			(2)	
(d)(i)	300000 to nearest hundred thousand		B1	3.4
(d)(ii)	Uses $200000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give t =		M1	3.4
	60.2 years to 3sf		A1ft	1.1b
			(3)	
(e)	Any two valid reasons- e.g. • 100 years is a long time and population may be affected by wars and disease • Inaccuracies in measuring gradient may result in widely different estimates • Population growth may not be proportional to population size • The model predicts unlimited growth		B2	3.5b
			(2)	

Question 14 continued
Notes:
(a) **M1:** Uses a linear equation to relate log P and t **A1:** Correct use of gradient and intercept to give a correct line equation
(b) **M1:** **Way 1:** Uses logs correctly to give log equation; **Way 2:** Uses powers correctly to "undo" log equation and expresses as product of two powers **M1:** **Way 1:** Identifies log b or log a or both; **Way 2:** Identifies a or b as powers of 10 **A1:** Correct value for a or b **A1:** Correct values for both
(c)(i) **B1:** Accept equivalent answers e.g. The population at $t = 0$ **(c)(ii)** **B1:** So accept rate at which the population is increasing each year or scale factor 1.01 or increase of 1% per year
(d)(i) **B1:** cao **(d)(ii)** **M1:** As in the scheme **A1ft:** On their values of a and b with correct log work
(e) **B2:** As given in the scheme – any two valid reasons

Question	Scheme	Marks	AOs
15	Finds $\frac{\mathrm{d}y}{\mathrm{d}x} = 8x - 6$	M1	3.1a
	Gradient of curve at P is –2	M1	1.1b
	Normal gradient is $-\frac{1}{m} = \frac{1}{2}$	M1	1.1b
	So equation of normal is $(y-2) = \frac{1}{2}\left(x - \frac{1}{2}\right)$ or $4y = 2x+7$	A1	1.1b
	Eliminates y between $y = \frac{1}{2}x + \ln(2x)$ and their normal equation to give an equation in x	M1	3.1a
	Solves their $\ln 2x = \frac{7}{4}$ so $x = \frac{1}{2}\mathrm{e}^{\frac{7}{4}}$	M1	1.1b
	Substitutes to give value for y	M1	1.1b
	Point Q is $\left(\frac{1}{2}\mathrm{e}^{\frac{7}{4}}, \frac{1}{4}\mathrm{e}^{\frac{7}{4}} + \frac{7}{4}\right)$	A1	1.1b
		(8 marks)	

Notes:

M1: Differentiates correctly

M1: Substitutes $x = \frac{1}{2}$ to find gradient (may make a slip)

M1: Uses negative reciprocal gradient

A1: Correct equation for normal

M1: Attempts to eliminate y to find an equation in x

M1: Attempts to solve their equation using exp

M1: Uses their x value to find y

A1: Any correct exact form

Question	Scheme	Marks	AOs
16(a)	Sets $2xy + \frac{\pi x^2}{2} = 250$	B1	2.1
	Obtain $y = \frac{250 - \frac{\pi x^2}{2}}{2x}$ and substitute into P	M1	1.1b
	Use $P = 2x + 2y + \pi x$ with their y substituted	M1	2.1
	$P = 2x + \frac{250}{x} - \frac{\pi x^2}{2x} + \pi x = 2x + \frac{250}{x} + \frac{\pi x}{2}$ *	A1*	1.1b
		(4)	
(b)	$x > 0$ and $y > 0$ (distance) $\Rightarrow \frac{250 - \frac{\pi x^2}{2}}{2x} > 0$ or $250 - \frac{\pi x^2}{2} > 0$ o.e.	M1	2.4
	As x and y are distances they are positive so $0 < x < \sqrt{\frac{500}{\pi}}$ *	A1*	3.2a
		(2)	
(c)	Differentiates P with negative index correct in $\frac{\mathrm{d}P}{\mathrm{d}x}$; $x^{-1} \to x^{-2}$	M1	3.4
	$\frac{\mathrm{d}P}{\mathrm{d}x} = 2 - \frac{250}{x^2} + \frac{\pi}{2}$	A1	1.1b
	Sets $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$ and proceeds to $x =$	M1	1.1b
	Substitutes their x into $P = 2x + \frac{250}{x} + \frac{\pi x}{2}$ to give perimeter = 59.8 M	A1	1.1b
		(4)	
			(10 marks)

Question 16 continued
Notes:
(a) **B1:** Correct area equation **M1:** Rearranges **their** area equation to make y the subject of the formula and attempt to use with an expression for P **M1:** Use correct equation for perimeter with their y substituted **A1*:** Completely correct solution to obtain and state printed answer
(b) **M1:** States $x > 0$ and $y > 0$ and uses their expression from (a) to form inequality **A1*:** Explains that x and y are positive because they are distances, and uses correct expression for y to give the printed answer correctly
(c) **M1:** Attempt to differentiate P (deals with negative power of x correctly) **A1:** Correct differentiation **M1:** Sets derived function equal to zero and obtains $x =$ **A1:** The value of x may not be seen (it is 8.37 to 3sf or $\sqrt{\left(\frac{500}{4+\pi}\right)}$) Need to see awrt 59.8 M with units included for the perimeter

<table>
<tr><th>Question</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">17 (a)</td><td><u>Way 1:</u>
Finds circle equation
$(x \pm 2)^2 + (y \mp 6)^2 =$
$(10 \pm (-2))^2 + (11 \mp 6)^2$</td><td><u>Way 2:</u>
Finds distance between
$(-2, 6)$ and $(10, 11)$</td><td>M1</td><td>3.1a</td></tr>
<tr><td>Checks whether (10, 1) satisfies their circle equation</td><td>Finds distance between $(-2, 6)$ and $(10, 1)$</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Obtains
$(x+2)^2 + (y-6)^2 = 13^2$
and checks that
$(10+2)^2 + (1-6)^2 = 13^2$ so states that (10, 1) lies on C *</td><td>Concludes that as distance is the same (10, 1) lies on the circle C *</td><td>A1*</td><td>2.1</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td></tr>
<tr><td rowspan="8">(b)</td><td colspan="2">Finds radius gradient $\dfrac{11-6}{10-(-2)}$ or $\dfrac{1-6}{10-(-2)}$ (m)</td><td>M1</td><td>3.1a</td></tr>
<tr><td colspan="2">Finds gradient perpendicular to their radius using $-\dfrac{1}{m}$</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">Finds (equation and) y intercept of tangent (see note below)</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">Obtains a correct value for y intercept of their tangent i.e. 35 or –23</td><td>A1</td><td>1.1b</td></tr>
<tr><td><u>Way 1</u>: Deduces gradient of second tangent</td><td><u>Way 2</u>: Deduces midpoint of PQ from symmetry (0, 6)</td><td>M1</td><td>1.1b</td></tr>
<tr><td>Finds (equation and) y intercept of second tangent</td><td>Uses this to find other intercept</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">So obtains distance PQ = 35 + 23= 58*</td><td>A1*</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(7)</td><td></td></tr>
<tr><td colspan="5" align="right">(10 marks)</td></tr>
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Question 17 continued

Notes:

(a) <u>Way 1</u> and <u>Way 2</u>:

M1: Starts to use information in question to find equation of circle or radius of circle
M1: Completes method for checking that (10, 1) lies on circle
A1*: Completely correct explanation with no errors concluding with statement that circle passes through (10, 1)

(b)

M1: Calculates $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)

M1: Finds $-\frac{1}{m}$ (correct answer is $-\frac{12}{5}$ **or** $\frac{12}{5}$). This is referred to as m' in the next note

M1: Attempts $y-11 = their\left(-\frac{12}{5}\right)(x-10)$ or $y-1 = their\left(\frac{12}{5}\right)(x-10)$ and puts $x = 0$, or uses vectors to find intercept e.g. $\frac{y-11}{10} = -m'$

A1: One correct intercept 35 or – 23

<u>Way 1</u>:

M1: Uses the negative of their previous tangent gradient or uses a correct $-\frac{12}{5}$ or $\frac{12}{5}$

M1: Attempts the second tangent equation and puts $x = 0$ or uses vectors to find intercept e.g. $\frac{11-y}{10} = m'$

<u>Way 2</u>:

M1: Finds midpoint of PQ from symmetry. (This is at (0, 6))
M1: Uses this midpoint to find second intercept or to find difference between midpoint and first intercept. e.g. 35 – 6 = 29 then 6 – 29 = –23 so second intercept is at (–23, 0)

<u>Ways 1 and 2</u>:

A1*: Obtain 58 correctly from a valid method