Accredited

Oxford Cambridge and RSA

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Sample Question Paper

Date – Morning/Afternoon

Version 2

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r|<1$

Binomial series

$(a+b)^n = a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + \ldots + {}^nC_r\, a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$,

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad \left(|x|<1,\ n \in \mathbb{R}\right)$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h\to 0} \frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

Integration

$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\int \mathrm{f}'(x)\left(\mathrm{f}(x)\right)^n \mathrm{d}x = \frac{1}{n+1}\left(\mathrm{f}(x)\right)^{n+1} + c$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta, \cos\theta \approx 1-\frac{1}{2}\theta^2, \tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \tfrac{1}{2} h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\bar{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma f x^2}{\Sigma f} - \bar{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \le z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \tfrac{1}{2}at^2$

$s = \tfrac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \tfrac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \tfrac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics

Answer **all** the questions

1 **(a)** If $|x|=3$, find the possible values of $|2x-1|$. **[3]**

(b) Find the set of values of x for which $|2x-1|>x+1$.
Give your answer in set notation. **[4]**

2 **(a)** Use the trapezium rule, with four strips each of width 0.25, to find an approximate value for $\int_0^1 \frac{1}{\sqrt{1+x^2}}\,\mathrm{d}x$. **[3]**

(b) Explain how the trapezium rule might be used to give a better approximation to the integral given in part **(a)**. **[1]**

3 **In this question you must show detailed reasoning.**

Given that $5\sin 2x = 3\cos x$, where $0^\circ < x < 90^\circ$, find the exact value of $\sin x$. **[4]**

4 For a small angle θ, where θ is in radians, show that $1+\cos\theta-3\cos^2\theta \approx -1+\frac{5}{2}\theta^2$. **[4]**

5 **(a)** Find the first three terms in the expansion of $\left(1+px\right)^{\frac{1}{3}}$ in ascending powers of x. **[3]**

(b) The expansion of $\left(1+qx\right)\left(1+px\right)^{\frac{1}{3}}$ is $1+x-\frac{2}{9}x^2+\ldots$.

Find the possible values of the constants p and q. **[5]**

6 A curve has equation $y=x^2+kx-4x^{-1}$ where k is a constant.

Given that the curve has a minimum point when $x=-2$

- find the value of k
- show that the curve has a point of inflection which is not a stationary point. **[7]**

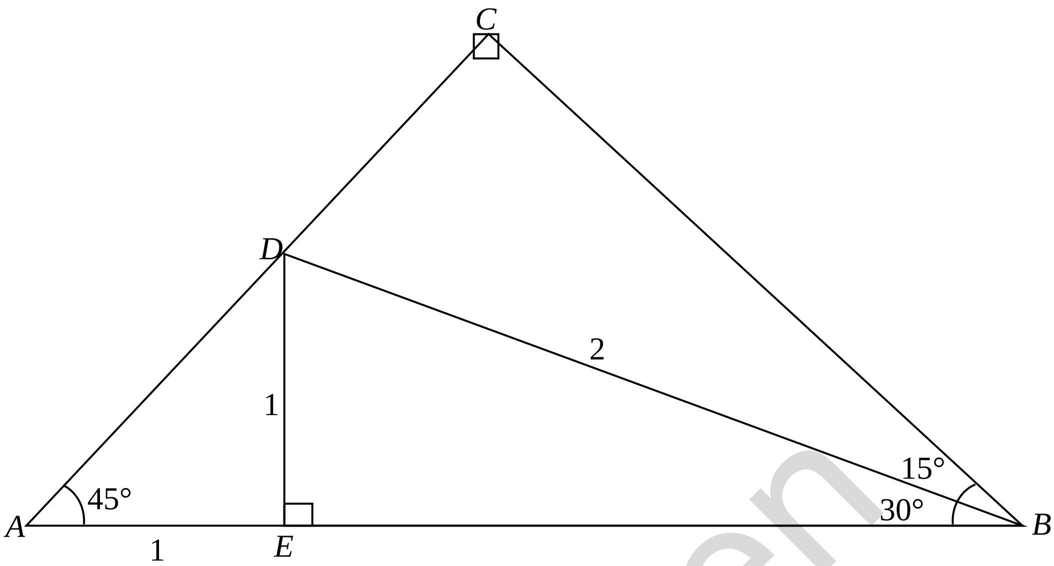
7 **(a)** Find $\int 5x^3\sqrt{x^2+1}\,\mathrm{d}x$. **[5]**

(b) Find $\int \theta\tan^2\theta\,\mathrm{d}\theta$.

You may use the result $\int \tan\theta\,\mathrm{d}\theta=\ln\left|\sec\theta\right|+c$. **[5]**

8 **In this question you must show detailed reasoning.**

The diagram shows triangle *ABC*.



The angles *CAB* and *ABC* are each 45°, and angle $ACB = 90^\circ$.
The points *D* and *E* lie on *AC* and *AB* respectively. $AE = DE = 1$, $DB = 2$.
Angle $BED = 90^\circ$, angle $EBD = 30^\circ$ and angle $DBC = 15^\circ$.

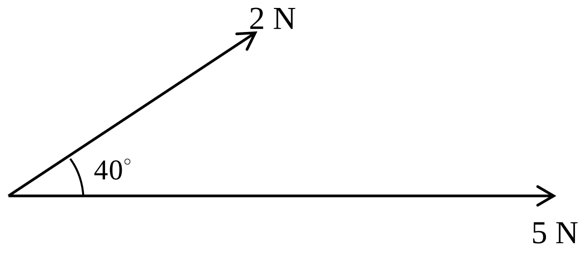
(a) Show that $BC = \dfrac{\sqrt{2}+\sqrt{6}}{2}$. **[3]**

(b) By considering triangle *BCD*, show that $\sin 15^\circ = \dfrac{\sqrt{6}-\sqrt{2}}{4}$. **[3]**

Section B: Mechanics

Answer **all** the questions

9 Two forces, of magnitudes 2 N and 5 N, act on a particle in the directions shown in the diagram below.



(a) Calculate the magnitude of the resultant force on the particle. **[3]**

(b) Calculate the angle between this resultant force and the force of magnitude 5 N. **[1]**

10 A body of mass 20 kg is on a rough plane inclined at angle α to the horizontal.
The body is held at rest on the plane by the action of a force of magnitude P N.
The force is acting up the plane in a direction parallel to a line of greatest slope of the plane.
The coefficient of friction between the body and the plane is μ.

(a) When $P = 100$, the body is on the point of sliding down the plane.

Show that $g \sin\alpha = g\mu\cos\alpha + 5$. **[4]**

(b) When P is increased to 150, the body is on the point of sliding up the plane.

Use this, and your answer to part **(a)**, to find an expression for α in terms of g. **[3]**

11 In this question the unit vectors **i** and **j** are in the directions east and north respectively.

A particle of mass 0.12 kg is moving so that its position vector **r** metres at time *t* seconds is given by $\mathbf{r} = 2t^3\mathbf{i} + \left(5t^2 - 4t\right)\mathbf{j}$.

(a) Show that when $t = 0.7$ the bearing on which the particle is moving is approximately 044°. **[3]**

(b) Find the magnitude of the resultant force acting on the particle at the instant when $t = 0.7$. **[4]**

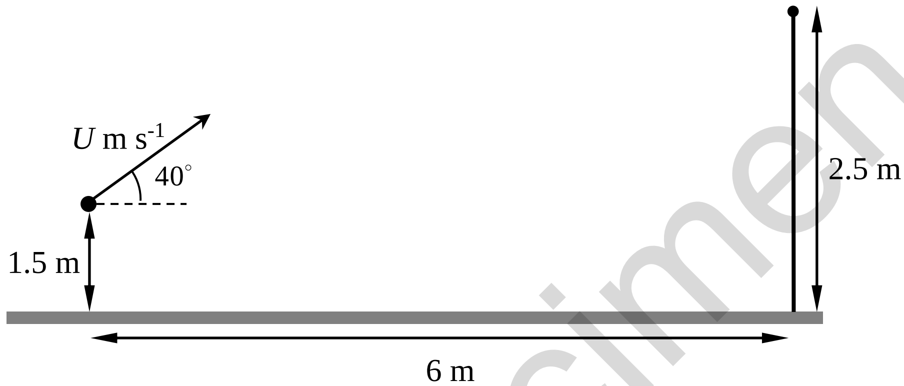
(c) Determine the times at which the particle is moving on a bearing of 045°. **[2]**

12 A girl is practising netball.
She throws the ball from a height of 1.5 m above horizontal ground and aims to get the ball through a hoop.
The hoop is 2.5 m vertically above the ground and is 6 m horizontally from the point of projection.

The situation is modelled as follows.

- The initial velocity of the ball has magnitude U m s^{-1}.
- The angle of projection is 40°.
- The ball is modelled as a particle.
- The hoop is modelled as a point.

This is shown on the diagram below.



(a) For $U = 10$, find

(i) the greatest height above the ground reached by the ball **[5]**

(ii) the distance between the ball and the hoop when the ball is vertically above the hoop. **[4]**

(b) Calculate the value of U which allows her to hit the hoop. **[3]**

(c) How appropriate is this model for predicting the path of the ball when it is thrown by the girl? **[1]**

(d) Suggest one improvement that might be made to this model. **[1]**

13 Particle A, of mass m kg, lies on the plane Π_1 inclined at an angle of $\tan^{-1}\frac{3}{4}$ to the horizontal.

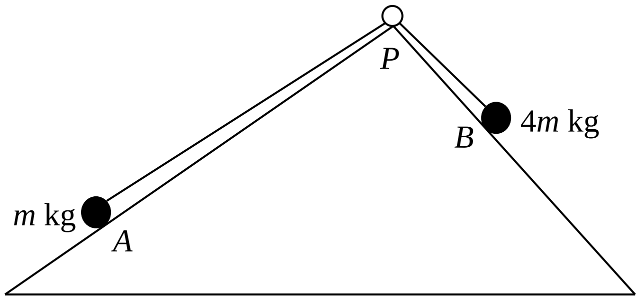
Particle B, of $4m$ kg, lies on the plane Π_2 inclined at an angle of $\tan^{-1}\frac{4}{3}$ to the horizontal.

The particles are attached to the ends of a light inextensible string which passes over a smooth pulley at P.

The coefficient of friction between particle A and Π_1 is $\frac{1}{3}$ and plane Π_2 is smooth.

Particle A is initially held at rest such that the string is taut and lies in a line of greatest slope of each plane.

This is shown on the diagram below.



(a) Show that when A is released it accelerates towards the pulley at $\dfrac{7g}{15}$ m s^{-2}. **[6]**

(b) Assuming that A does not reach the pulley, show that it has moved a distance of $\frac{1}{4}$ m when its speed is $\sqrt{\dfrac{7g}{30}}$ m s^{-1}. **[2]**

14 A uniform ladder AB of mass 35 kg and length 7 m rests with its end A on rough horizontal ground and its end B against a rough vertical wall.

The ladder is inclined at an angle of 45° to the horizontal.

A man of mass 70 kg is standing on the ladder at a point C, which is x metres from A.

The coefficient of friction between the ladder and the wall is $\frac{1}{3}$ and the coefficient of friction between the ladder and the ground is $\frac{1}{2}$.

The system is in limiting equilibrium.

Find x. **[8]**

END OF QUESTION PAPER

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